

20 Minutes Revision Book

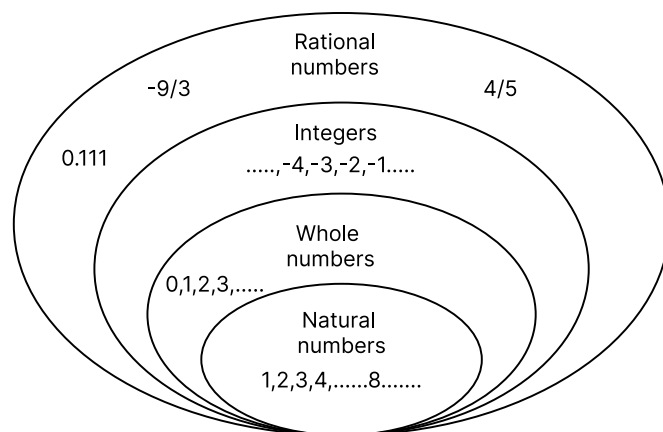
CBSE Class 10 Maths Mind Maps

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RATIONAL NUMBERS

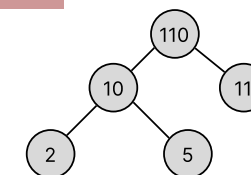
A number that can be expressed in the form of P/q where p and q are integers and $q \neq 0$ is called a rational number.



THE FUNDAMENTAL THEOREM OF ARITHMETIC (PRIME FACTORIZATION METHOD)

A composite number can be expressed (factorised) as a product of primes and this factorization is unique, apart from the order in which the prime factorization occurs.

Eg: $110 = 2 \times 5 \times 11$



Note: for two positive numbers a and b , $HCF(a, b) \times LCM(a, b) = a \times b$

Eg: $6 = 2^1 \times 3^1$ and $20 = 2^2 \times 5^1$

$LCM(6, 20) = 2^2 \times 3 \times 5 = 60$, $HCF(6, 20) = 2$
 $\Rightarrow HCF(6, 20) \times LCM(6, 20) = 6 \times 20$

Real
Numbers





HCF, LCM OF THREE INTEGERS

$\text{HCF}(p,q,r) \times \text{LCM}(p,q,r) \neq p \times q \times r$, where p, q, r are positive integers. However, the following results hold good for three numbers p, q and r

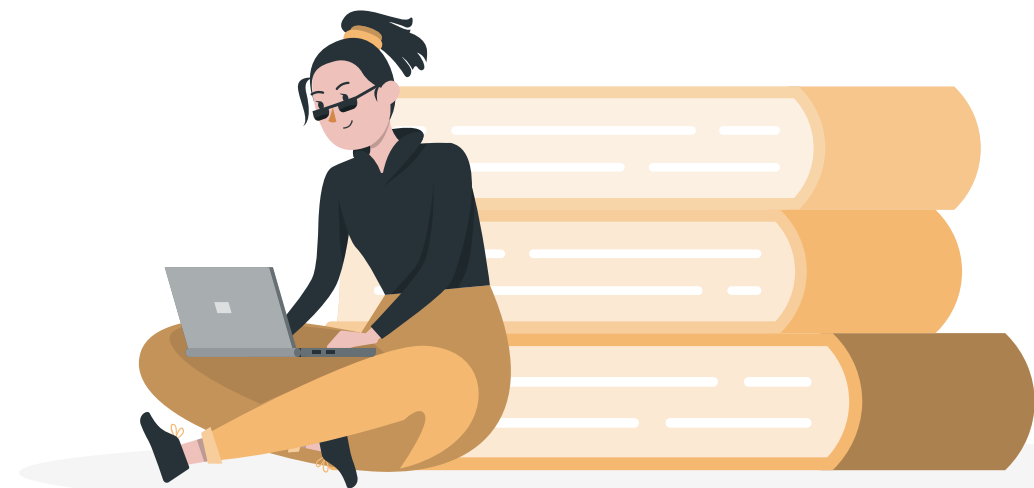
$$\text{LCM}(p,q,r) = \frac{p \cdot q \cdot r \cdot \text{HCF}(p,q,r)}{\text{HCF}(p,q) \cdot \text{HCF}(q,r) \cdot \text{HCF}(p,r)}$$

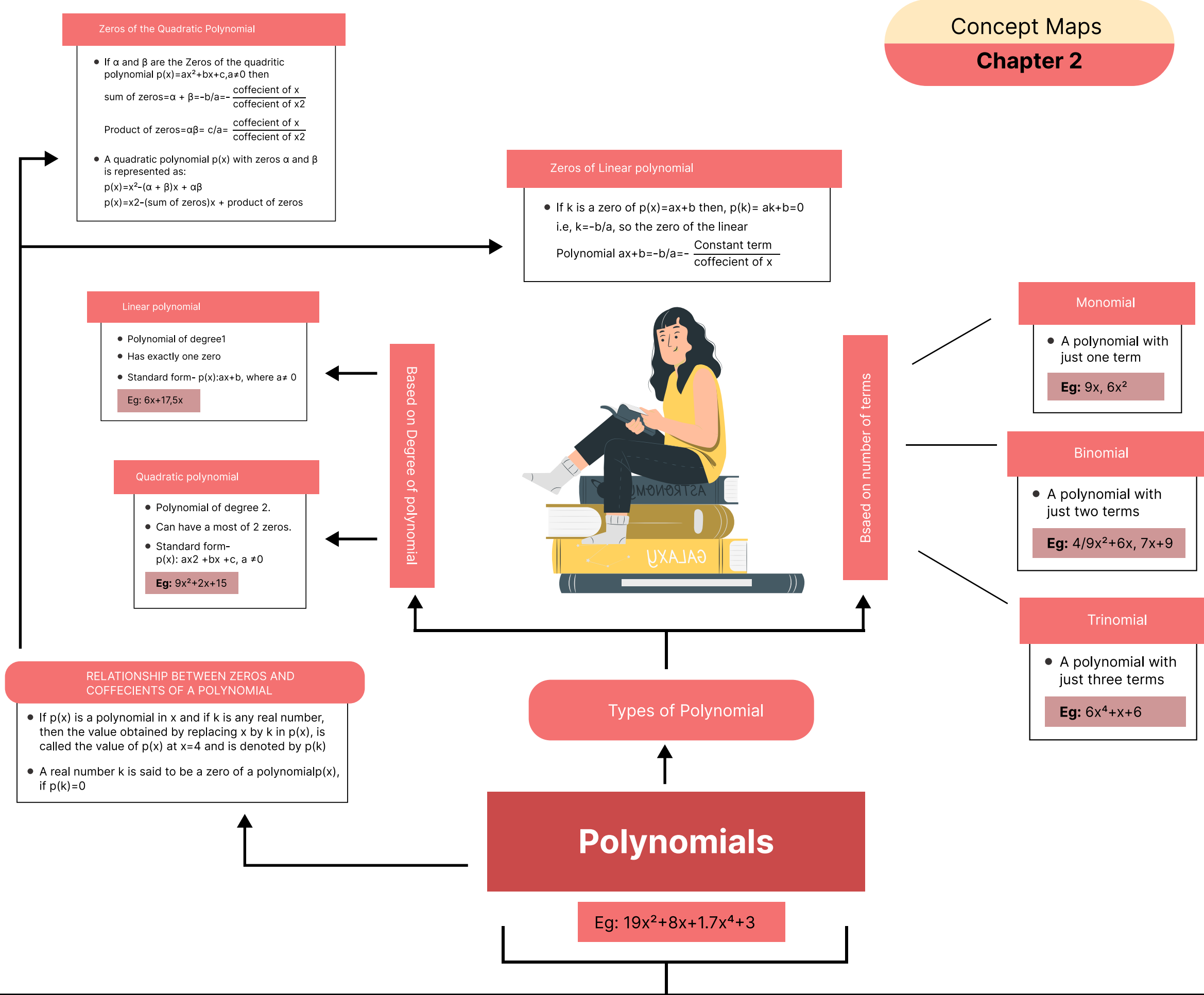
$$\text{HCF}(p,q,r) = \frac{p \cdot q \cdot r \cdot \text{LCM}(p,q,r)}{\text{LCM}(p,q) \cdot \text{LCM}(q,r) \cdot \text{LCM}(p,r)}$$

IRRATIONAL Numbers

A number which cannot be expressed in the form of p/q where p and q are integers and $q \neq 0$ is called irrational numbers

Eg: $\sqrt{2}, 5 + \sqrt{3}, \sqrt{7}-4, 0.8572984\dots, 0.10100100010\dots, \pi$, etc.





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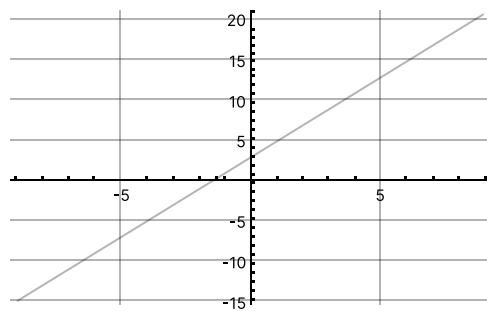


Graph of a polynomial

Geometrical Representation of a linear polynomial

- Graph of a linear Polynomial $p(x)=ax+b$ is a straight line. It cuts the x-axis at exactly one point.
- In the polynomial $p(x)=ax+b$ for degree 1, a represents the slope of a line $ax+b$, the constant b represents y-intercept of a line.

Eg: $y=2x+3$
Here $a=2$ and $b=3$

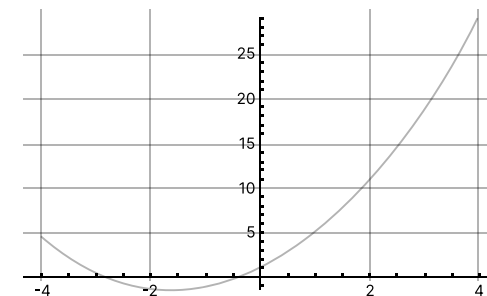


Note: All constant functions are linear

Geometrical Representation of a Quadratic polynomial

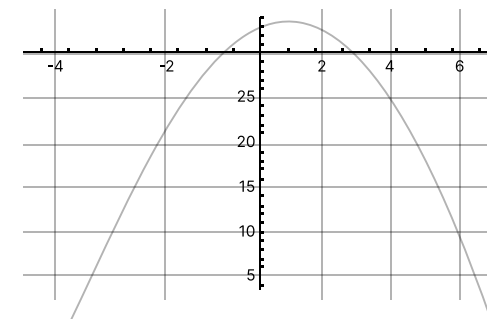
- Graph of Quadratic Polynomial $p(x)=ax^2+bx+c$ is a parabola open upwards, if $a>0$.

Eg: $y=x^2+3x+1$

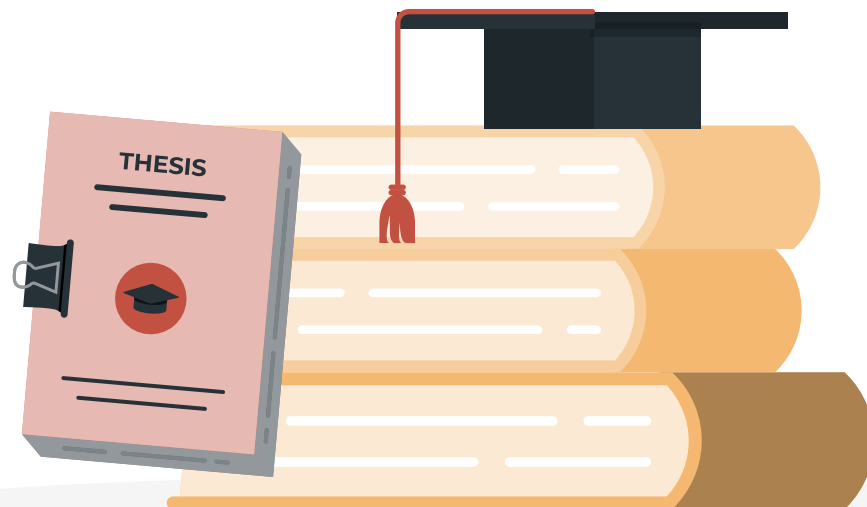


- Graph of Quadratic Polynomial $p(x)=ax^2+bx+c$ is a parabola open downwards, if $a<0$.

Eg: $y=-x^2+3x+1$



Note: A polynomial $p(x)$ of degree n has at most n zeros



Concept Maps

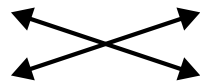
Chapter 3



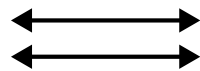
NATURE OF TWO STRAIGHT LINES IN A PLANE

If two lines are in plane, then only one of the following three possibilities can happen:

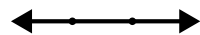
(i) The two lines will intersect at one point.



(ii) The two lines will not intersect, that means the lines are parallel



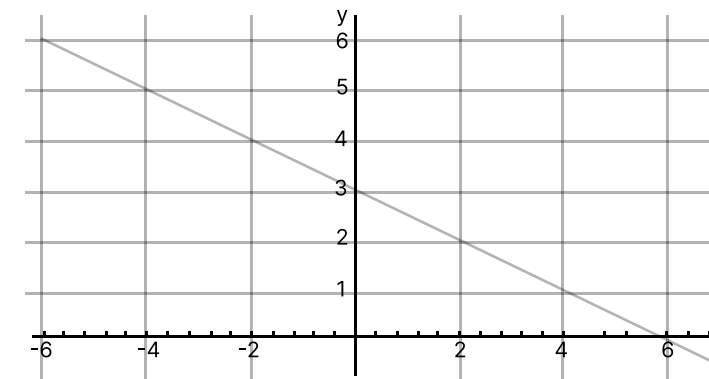
(iii) The two lines will be coincident.



GRAPHICAL REPRESENTATION OF A LINEAR EQUATION

Graphical representation of a linear equation in two variables is a straight line

Eg: $x + 2y - 6 = 0$



LINEAR EQUATION

- An equation in the form $ax + by + c = 0$, where a, b and c are real numbers, a and b are not both zero
- Each solution (x, y) of a linear equation in two variables, $ax + by + c = 0$, corresponds to a point on the line representing the equation, and vice versa.



REPRESENTATION OF A PAIR OF LINEAR EQUATION

The general form of a pair of linear equation is
 $a_1x + b_1y + c_1 = 0$
 $a_2x + b_2y + c_2 = 0$



GENERAL FORM OF A PAIR OF LINEAR EQUATION IN TWO VARIABLES

The general form of a pair of linear equation in two variables x and y is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where a_1, b_1, c_1 and a_2, b_2, c_2 are all real numbers and $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$

Pair of Linear Equation in two variables



Solution of a Pair of Linear Equation in Two Variables

The solution of a linear equation in two variables 'x' and 'y' is a pair of values which makes the two sides of the equation equal.

Identification of the type of pair of linear equation using coefficients

- (i) If $a_1/a_2 \neq b_1/b_2$, then the pair of linear equation is consistent and has an unique solution
- (ii) If $a_1/a_2 = b_1/b_2 = c_1/c_2$, then the pair of linear equation is consistent and has infinite many solutions.
- (iii) If $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, then the pair of linear equation is inconsistent and has no solution.

Graphical Method

if a pair of linear equation is given by
 $a_1x + b_1y + c_1 = 0$
 $a_2x + b_2y + c_2 = 0$

Algebraic method

Substitution method

Eg:
Solve the following system by substitution

$$4x + 7y = 5$$

$$x + y = 5$$

Sol: Let $4x + 7y = 5 \dots (1)$

$$\text{And, } x + y = 5 \dots (2)$$
$$y = 5 - x \dots (3)$$

Substituting the value of y in eq. (1)

$$4x + 7y = 5$$

$$4x + 7(5 - x) = 5$$

$$4x + 35 - 7x = 5$$

$$-3x = 5 - 35$$

$$-3x = -30$$

$$x = 10$$

Now put the value of x in eq. (3)

$$y = 5 - x$$

$$y = 5 - 10 = -5$$

The solution is (x, y) = (10, -5)

Elimination Method

Eg:
Solve the following system by elimination method

$$2x + 3y = 8 \dots (1)$$

$$3x + 2y = 7 \dots (2)$$

Sol: On multiplying the eq. (1) by 3 and eq (2) by 2

$$3x \text{ (eq.1)} \rightarrow 3x(2x + 3y = 8) \rightarrow 6x + 9y = 24 \dots (3)$$

$$2x \text{ (eq.2)} \rightarrow 2x(3x + 2y = 7) \rightarrow 6x + 4y = 14 \dots (4)$$

Subtract the eq (4) from the eq. (3)

$$6x + 9y = 24$$

$$-(6x + 4y = 14)$$

$$5y = 10$$

$$y = 2 \quad (\text{eliminating } x)$$

Put the value of y in eq (1)

$$2x + 3(2) = 8$$

$$2x + 6 = 8$$

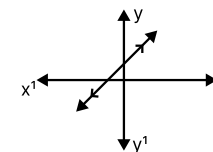
$$2x = 2$$

$$x = 1$$

Solution: $x = 1, y = 2$ or (1, 2).

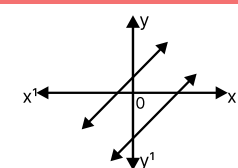
(eliminating x)

Infinitely many solutions



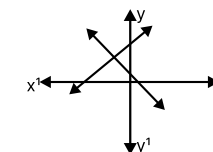
- The two equations of lines coincident over one other.
- Every point on the lines is the solution. Hence infinitely many solutions are there.

No solutions



- The two equations of lines are parallel to each other.
- Since they never intersect, so they have no solution.

Infinitely many solutions



- The two equations of lines intersect at a point. The point of intersection is the solution of two equations.



Concept Maps

Chapter 4



NATURE OF ROOTS

- For a quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, the expression $b^2 - 4ac$ is called the discriminant of the quadratic equation.
- The discriminant determines the nature of roots of the quadratic equation based on the coefficients of the quadratic polynomial.
 - (i) Two distinct real roots, if $b^2 - 4ac > 0$,
 - (ii) Two equal real roots, if $b^2 - 4ac = 0$,
 - (iii) No real roots, if $b^2 - 4ac < 0$.



QUADRATIC EQUATION

- An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$ is called a quadratic equation in one variable x , where a, b, c are constants.
- Standard form: $ax^2 + bx + c = 0$, $a \neq 0$.

Eg: $21x^2 + 3x + 8 = 0$

Quadratic Equations

Quadratic formula

- The Quadratic Formula of equation $ax^2 + bx + c = 0$ in terms of x is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- If $b^2 - 4ac > 0$, then roots are real and are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac < 0$, then equation will have no real roots.
- If $b^2 - 4ac = 0$, then equation will have 2 equal roots and are $\frac{-b}{2a}$ and $\frac{-b}{2a}$

Find the roots of the equation $2x^2 + x - 4 = 0$, by quadratic formula.

Sol: On comparing this equation by $ax^2 + bx + c = 0$ we obtain $a = 2$, $b = 1$ and $c = -4$

By using Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1 + 4 \times 2 \times (-4)}}{2 \times 2}$$

$$x = \frac{-1 \pm \sqrt{33}}{4}$$

Therefore $x = \frac{-1 + \sqrt{33}}{4}$ or $x = \frac{-1 - \sqrt{33}}{4}$

Factorization method

- A real number α is called root of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$

Note: The zeroes of quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.

Eg:

Find the roots of the equation $x^2 - 3x - 10 = 0$, by factorization.

Sol: $x^2 - 3x - 10 = 0$

On splitting the middle term

$$x^2 - 5x + 2x - 10 = 0$$

$$x(x - 5) + 2(x - 5) = 0$$

$$(x + 2)(x - 5) = 0$$

Thus, $x = -2$ and $x = 5$ are the roots of the given quadratic equation.

This method of solving a quadratic equation is called the factorization method



Solution of a Quadratic Equation





Arithmetic Progression

- A sequence of numbers in which difference between consecutive terms is constant is called AP.
- Common difference is a difference between two consecutive numbers in an AP.
- Common difference can be positive, negative or zero
 - If it is Positive, then AP is increasing.
 - If it is Negative, then AP is decreasing.
 - If common difference is zero, then AP is constant.

General form of an AP: $a+d, a+2d, a+3d, \dots$

Where 'a' is first term and 'd' is common difference.

Eg:

Given AP is 3, 5, 7, 9,

Here, a = 3

and $d = 5 - 3 = 7 - 5 = 9 - 7 = 2$

First term is $a = 3$

Second term is $a_2 = a + d = 3 + 2 = 5$

Third term is $a_3 = a + 2d = 3 + 2(2) = 7$ and so on,

General term: If 'a' is the first term and 'd' is common difference in an A.P., then n^{th} term is given by $a_n = a + (n-1)d$ where $a_n = n^{\text{th}}$ term (General term).

Eg: 1, 2, 3, 4, ...

Term: Each number in the sequence is called a term.

FINITE ARITHMETIC PROGRESSION

When there are only a limited number of terms in a sequence.

Eg: 22, 24, 26, 28, ..., 34

INFINITE ARITHMETIC PROGRESSION

When there are an infinite number of terms in a sequence.

Eg: 3, 6, 9, 12, ...

Note: A finite AP will have the last term, Whereas infinite AP don't



Finite or infinite arithmetic progression

Concept Maps

Chapter 5



n^{th} TERM OF AN AP

- The n^{th} term, a_n of an AP with first term a and the common difference d is given by $a_n = a + (n-1)d$.
- a_n is also called the general term of the AP.
- a_n represents the last term which is denoted by T_n .

Eg:

Find the 7th term of the AP: 2, 7, 12, ...

Here $a = 2$, $d = 7 - 2 = 5$ and $n = 7$.

We have $a_n = a + (n-1)d$

so $a_7 = 2 + (7-1) \times 5 = 2 + 30 = 32$

Note:

- If a_n is given then common difference $d = a_n - a_{n-1}$
- If a, b, c are in AP, then $b = \frac{a+c}{2}$ and b is called arithmetic mean of a and c

Arithmetic Progression



SUM OF FIRST n^{th} TERMS OF AN AP

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

where

a = first term

d = common difference

n = total no of terms



IMPORTANT TERMS

Sequence- It is a collection of numbers, arranged in a definite order, according to some different rule.

Eg: 2, 4, 6, 8

Progression- A sequence in which the general term can be expressed using a mathematical progression.

Series- It is the sum of elements in the corresponding sequence

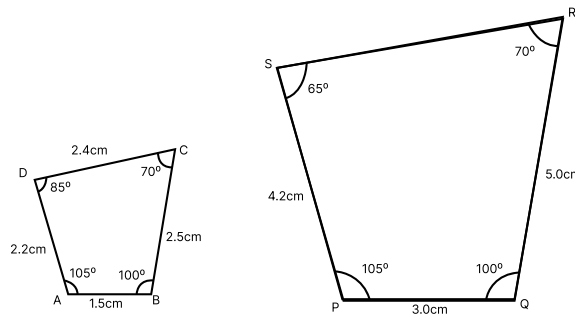
Eg: $1+2+3+4+5$ is the series of natural numbers

Concept Maps

Chapter 6

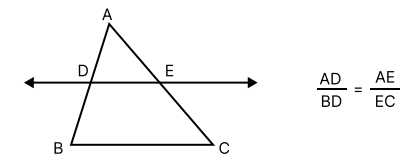
Similar Polygons

- Two polygons are said to be similar to each other, if:
 - Their corresponding angles are equal.
 - The length of their corresponding sides are proportional.



BASICS PROPORTIONALITY THEROM

- Theorem:** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



And its converse is also true.

When a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side
In $\triangle ABC$, a line DE intersecting AB at D and AC at E

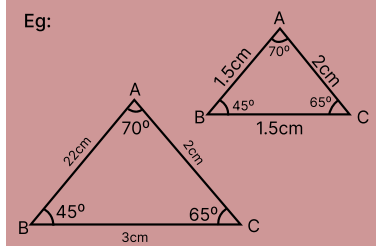


Triangles

SIMILAR FIGURES

- Figures having the same shape but not necessary the same size.

Eg:

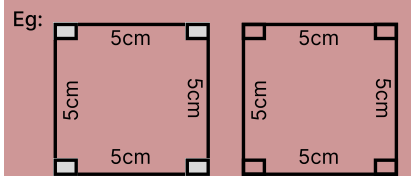


Note: All congruent figures are similar but the similar figures need to be congruent.

CONGRUENT FIGURES

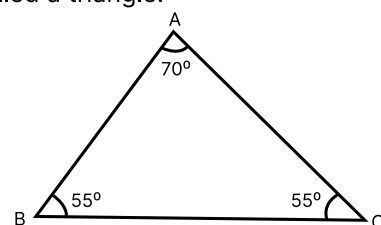
- Figures whose shapes and sizes are same or equal in all aspects.

Eg:



Triangle

- A simple closed curve or a polygon formed by three line-segments is called a triangle.



SIMILARITY OF TRIANGLES

- Two triangles are similar if,
 - Their corresponding angles are equal.
 - Their corresponding sides are in the same ratio (or proportion).

- Equiangular Triangles:** when corresponding angles of two triangles are equal, then they are known as equiangular triangles.

Note: The ratio of any two corresponding sides in two equiangular triangles is always the same.



CRITERIA FOR SIMILARITY OF TRIANGLE

- Two triangles are similar if either of the following three criterion's are satisfied.

AAA SIMILARITY CREATION

- Theorem:** If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.

In $\triangle ABC$ and $\triangle DEF$
 $\triangle ABC \sim \triangle DEF$ When $\angle A = \angle D = \angle B = \angle E$
 and $\angle C = \angle F$

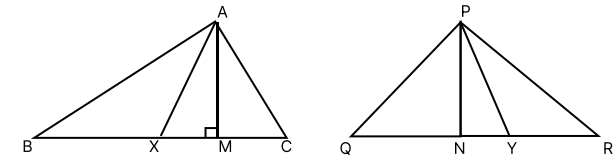
SAS SIMILARITY CREATION

- Theorem:** If one angle of a triangle to one angle of the other triangle a sides including these angles are proportional, then the two triangles similar.

In $\triangle ABC$ and $\triangle DEF$
 $\triangle ABC \sim \triangle DEF$ When $\frac{AB}{DE} = \frac{BC}{EF}$ = and $\angle B = \angle E$

PROPERTIES OF SIMILARITY OF TRIANGLE

- If two triangles, say $\triangle ABC$ and $\triangle PQR$ are similar, then



$$(i) \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle PQR} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AX}{PY} = \frac{AM}{PN}$$

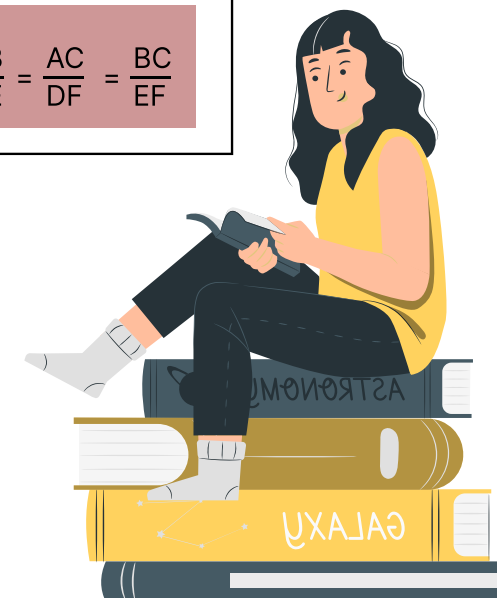
$$(ii) \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} = \frac{AX^2}{PY^2} = \frac{AM^2}{PN^2}$$

Where AX, AM and Py, PN are medians, altitudes of $\triangle ABC$ and $\triangle PQR$ respectively

SSS SIMILARITY CREATION

- Theorem:** If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

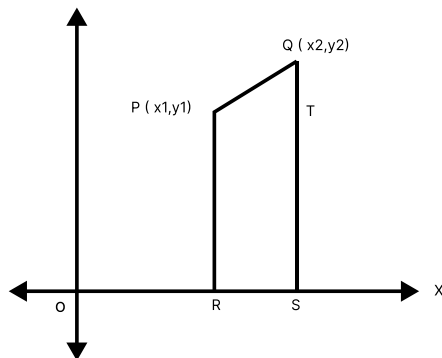
In $\triangle ABC$ and $\triangle DEF$
 $\triangle ABC \sim \triangle DEF$ When $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$



Concept Maps

Chapter 7

Distance formula



- Distance between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by:

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Which is called the distance formula

$$\text{We can also write, } PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Note: Distance can't be negative so we take positive square root

Eg:

Find the distance between the (2,3), (4,1) pair of points.

Sol: Let the two points be $P(2,3)$ and $Q(4,1)$

We need to find distance PQ ,

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here $x_1 = x$ coordinate of $P = 2$

$y_1 = y$ coordinate of $P = 3$

$x_2 = x$ coordinate of $Q = 4$

$y_2 = y$ coordinate of $Q = 1$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(4-2)^2 + (1-3)^2}$$

$$\sqrt{(2)^2 + (-2)^2}$$

$$\sqrt{4+4} = \sqrt{8}$$

Hence distance between two points is $\sqrt{8}$ Units.

Distance from Origin

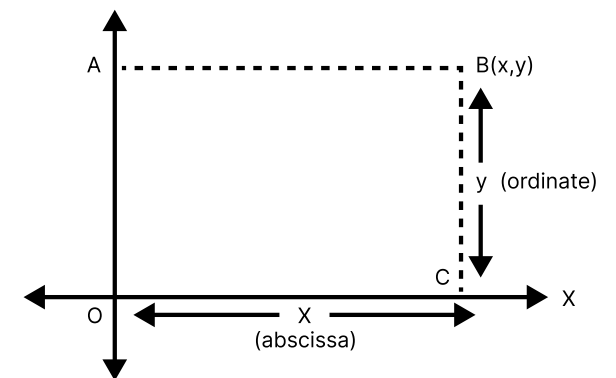
- Distance from the origin: The distance of the point $P(x, y)$ from the origin $O(0, 0)$ is given by:

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

Coordinate Geometry

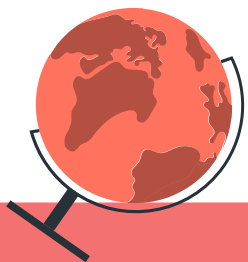
Coordinate Geometry

- Abscissa:** The distance of a point (x, y) from the y -axis is called x -coordinate or abscissa.
- Ordinate:** The distance of a point (x, y) from the x -axis is called y -coordinate or ordinate.



Note: The x -axis and y -axis are mutually perpendicular to each other.



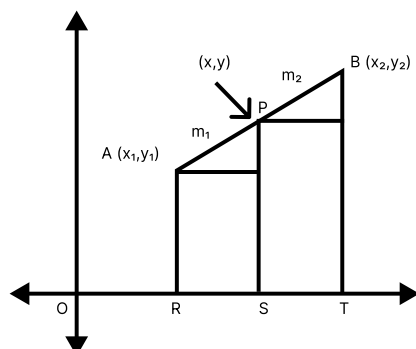


SECTION FORMULA

- The coordinates of the point $P(x,y)$ which divides the line segment joining the points $A(x_1,y_1)$ and $B(x_2,y_2)$ internally in the ratio

$$m_1 : m_2 \text{ are } \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

This is known as the section formula.



If the ratio in which P divides AB is $k:1$, then the coordinates of the point P

$$\text{will be } \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1} \right)$$

Eg:

Find the coordinates of the point which divides the join of $(-1,7)$ and $(4,-3)$ in the ratio $2:3$

Sol: Let the given points be $A(-1,7)$ and $B(4,-3)$

Let the point be $P(x,y)$ which divides AB in the ratio $2:3$

$$\text{Here, } x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

Where $m_1=2$, $m_2=3$

Where $m_1=2$, $m_2=3$

$$x_1=-1, x_2=4$$

$$y_1=7, y_2=-3$$

Hence, putting value

Hence, putting value

$$x = \frac{2 \times 4 + 3 \times (-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

$$y = \frac{2 \times (-3) + 3 \times 7}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

Hence $x=1$ and $y=3$

So, the required point is $P(x,y)=P(1,3)$



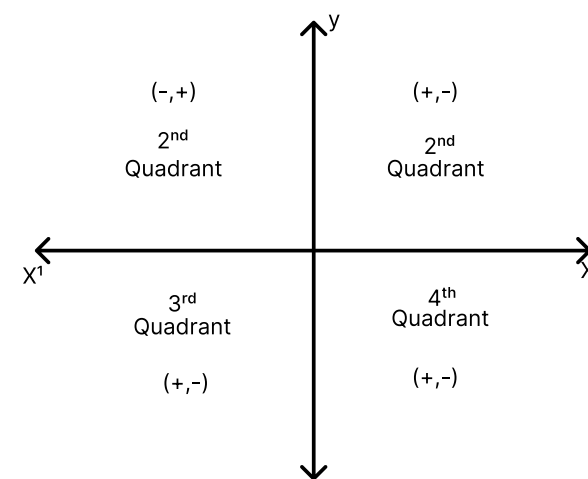
MIDPOINT FORMULA

The mid-point of a line segment divides the line segment in the ratio $1:1$. Therefore, the coordinates of the mid-point P of the join of the points $A(x_1,y_1)$ and $B(x_2,y_2)$ is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$



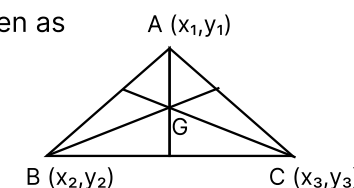
STUDY OF ALGEBRIC EQUATIONS ON GRAPH



CENTROID OF TRIANGLE

If $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are the coordinates of vertices of $\triangle ABC$, then the Coordinates of centroid G of the triangle is given as

$$G(x,y) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3} \right)$$



INTRODUCTION TO TRIGNOMETRY

-The trigonometric ratios of an acute angle in a right-angle triangle express the relationship between the angle and the length of its sides.

The trigonometric ratios of the angle A in right triangle ABC are defined as:

$$\text{sine of angle A} = \sin A = \frac{\text{Side opposite to angle A}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\text{sine of angle A} = \cos A = \frac{\text{Side Adjacent to angle A}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\text{sine of angle A} = \tan A = \frac{\text{Side opposite to angle A}}{\text{Side adjacent to angle A}} = \frac{BC}{AB}$$

cosecant of angle A = cosec A

$$= \frac{1}{\text{Side of angle A}} = \frac{\text{Hypotenuse}}{\text{Side opposite to angle A}} = \frac{AC}{BC}$$

secant of angle A = sec A

$$= \frac{1}{\text{cosine of angle A}} = \frac{\text{Hypotenuse}}{\text{Side adjacent to angle A}} = \frac{AC}{AB}$$

contangent of angle A = cot A

$$= \frac{1}{\text{cosine of angle A}} = \frac{\text{Side adjacent to angle A}}{\text{Side opposite to angle A}} = \frac{AB}{BC}$$

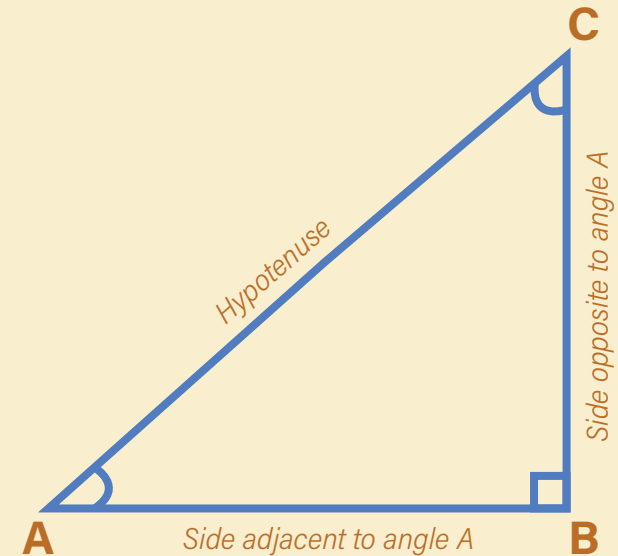
Note :

-The ratios cosec A, sec A, and cot A are the reciprocals of the ratios sin A, cos A and tan A, respectively.

-The symbol sin A is used as an abbreviation for 'the sine of the angle A'. sin A is not the product of 'sin' and A. 'sin' separated from A has no meaning.

-The values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

Remark: Since the hypotenuse is the longest side in a right triangle, the value of sin A or cos A is always less than 1.



Eg:

If $\sin A = \frac{3}{4}$ Calculate $\cos A$ and $\tan A$.

Sol: Given, $\sin A = \frac{3}{4} = \frac{\text{perpendicular}}{\text{hypotenuse}}$

By Pythagoras theorem,

$$\text{Hypotenuse}^2 = \text{Base}^2 + \text{Perpendicular}^2$$

$$\text{So, Base}^2 = \text{Hypotenuse}^2 - \text{Perpendicular}^2$$

$$= 4^2 - 3^2 = 16 - 9 = 7$$

$$\text{Base} = \sqrt{7}$$

$$\text{Therefore, } \cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{\sqrt{7}}{4}$$

$$\text{And, } \tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{3}{\sqrt{7}}$$

TRIGNOMETRIC RATIOS OF SOME SPECIFIC ANGLES

The specific angles are 0° , 30° , 45° , 60° , 90° . These are given in the following table:

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos A$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan A$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined
$\operatorname{cosec} A$	not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec A$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	not defined
$\cot A$	not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0

TRIGNOMETRY IDENTITIES

$$\begin{aligned} &-\cos^2 A + \sin^2 A = 1 \\ &-1 + \tan^2 A = \sec^2 A, 0^\circ \leq A < 90^\circ \\ &-\cot^2 A + 1 = \operatorname{cosec}^2 A, 0^\circ < A \leq 90^\circ \end{aligned}$$

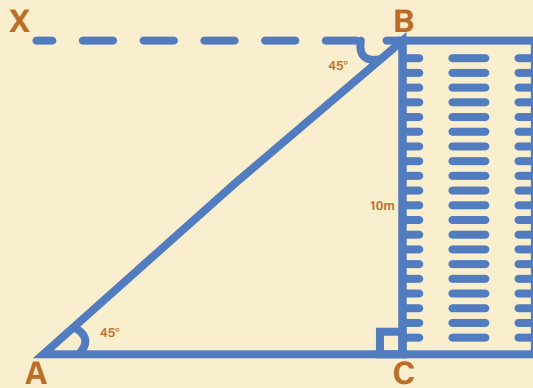
TRIGNOMETRY

-The study of relationships between the sides and angles of a triangle.

SOME APPLICATIONS OF TRIGNOMETRY

TRIGNOMETRIC IDENTITIES

Eg:
Suppose angle of depression from top of the tower to point A is 45° .
And height of tower is 10m. What is the distance of point A from the building?



Sol: Let the height of building be $BC = 10\text{m}$
And angle of depression from point is 45° .
BX is the horizontal Now, Ones BX and AC are parallel and AB is the transversal.
So Alternate angles are equal
 $\therefore \angle BAC = \angle XBA = 45^\circ$

So, $\tan A = \frac{\text{side opposite to angle A}}{\text{side adjacent to angle A}}$

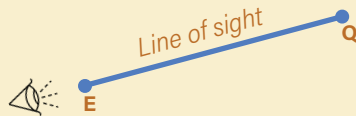
80
45.
11 AC
 $AC = 10\text{m}$ Therefore distance is 10m.

APPLICATIONS OF TRIGNOMETRY

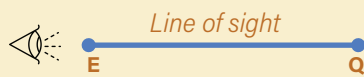
- To measure the height of a building or a mountain.
- Used in satellite systems.
- Used in oceanography to calculate heights of waves and tides in oceans.
- Used in navigating directions or creation of maps.

IMPORTANT TERMS

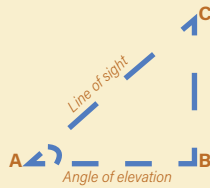
-Line of Sight: A line drawn from the eye of an Observer to the point in the object.



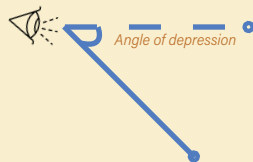
-Horizontal: The straight line between E and Q is called the horizontal line, if observer looks from a point E (eye) to another point Q which is horizontal



-Angle of Elevation: The angle formed by the line of sight with the horizontal plane for an object.



-Angle of Depression: If the point on the object is below the horizontal line, then the angle formed by the line of sight with the horizontal line is called the angle of depression.



ANGLE OF ELEVATION

Eg:

The angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of the tower, is 30° . Find the height of the tower.

Sol: Let AB is the length of tower

Let point be C

Base=30m

Hence, BC=30m

Angle of elevation = 30°

So $\angle ACB = 30^\circ$

Since tower is vertical.

$\angle ABC = 90^\circ$

In right triangle ABC

$$\tan C = \frac{\text{perpendicular}}{\text{base}}$$

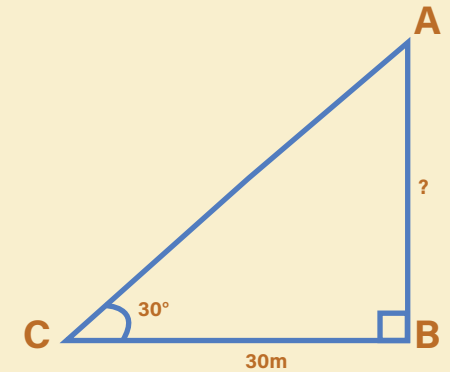
$$\tan 30^\circ = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{30}$$

$$AB = \frac{30}{\sqrt{3}}$$

$$AB = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$$

Hence, height of tower = AB = $10\sqrt{3}$ m



CIRCLE

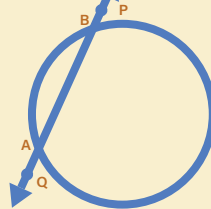
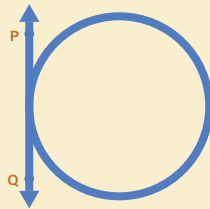
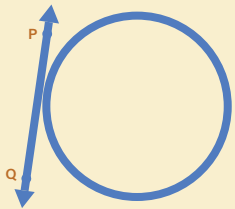
INTRODUCTION TO CIRCLES

-Circle: A Circle is a collection of all those points in a plane which are at a constant distance (radius) from a fixed point (centre).



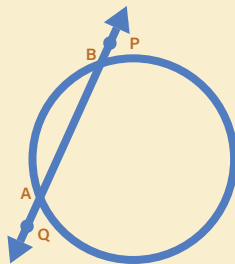
-For a circle and a line on a plane, there can be three possibilities.

- (i) They can be non-intersecting.
- (ii) They can have a single common point that means, the line touches the circle.
- (ii) They can have two common points that means, the line cuts the circle.



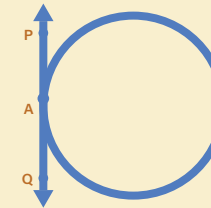
SECANT

-When a line intersects the circle in such a way that there are two common points then that line is called secant.



TANGENT

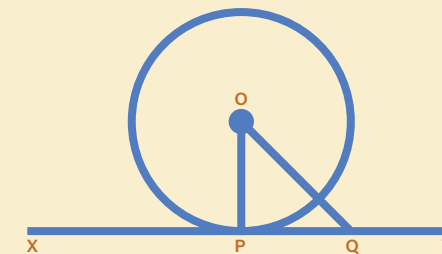
-When the line intersects the circle at only one point then, that line is known as tangent.



- The tangent to a circle is a special case of secant, when the two end points of corresponding chord coincide.
- There is no tangent to a circle passing through point lying inside the circle.
- There are exactly two tangents to a circle through a point outside the circle.

THEOREM 1

-The theorem states that "the tangent to the circle at any point is the perpendicular to the radius of the circle that passes through the point of contact".

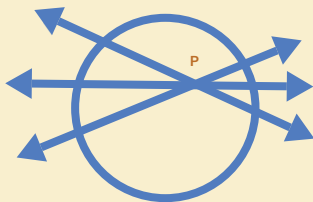


Here, O is the centre of the circle and OPLXY Note:

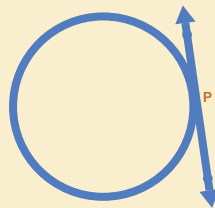
- There is one and only one tangent to a circle passing through a point lying on the circle.
- The line containing the radius through the point of contact is also called the 'normal' to the circle at the point.

NUMBER OF TANGENTS FROM A POINT O A CIRCLE

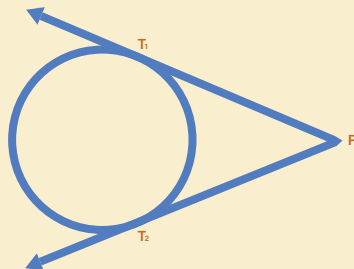
-There is no tangent to a circle passing through a point lying on the circle.



-There is one and only one tangent to a circle passing through a point lying on the circle.

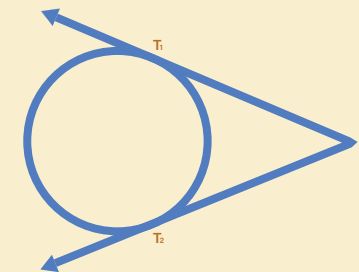


-There are exactly two tangents to a circle through a point lying outside the circle.



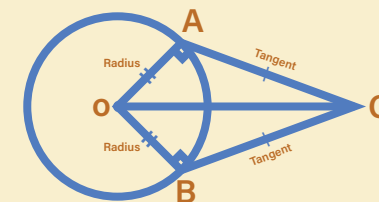
LENGTH OF A TANGENT

-The length of the segment of the tangent from the external point P and the point of contact with the circle i.e. T_1 or T_2



THEOREM 2

-The length of tangents drawn from an external point to a circle are equal.

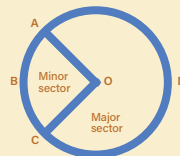


Here, two tangents are drawn from the external point C. As the tangents are perpendicular to the radius, they formed the right-angle triangle. So, $\triangle AOC$ and $\triangle BOC$ are congruent right triangles. Hence, $AC = BC$.

AREAS RELATED TO CIRCLE

Sector

-The area formed by an arc and the two radii joining the endpoints of the arc is called Sector.



Major sector

- The area including angle AOB with point D is called the major sector. So OADB is the major sector.

The angle of the major sector = $360^\circ - \text{angle AOB}$.
Area of major sector = Area of circle - Area of the Minor sector

Note: Area of minor sector + Area of the major sector = Area of the circle.

Minor sector

-The area including angle AOB with point C is called the minor sector. So OACB is the minor sector. Angle AOB is the angle of the minor sector.

$$\text{Area of the sector of angle } \theta = \frac{\theta}{360^\circ} \times \pi r^2$$

$$\text{Length of an arc of a sector of angle } \theta = \frac{\theta}{360^\circ} \times 2\pi r$$

Where r is the radius of the circle.

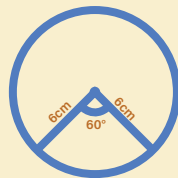
Eg.

-Find the area of a sector of a circle with radius 6cm if angle of the sector is 60° .

Sol: Given that, Radius = $r = 6\text{cm}$ And angle of the sector = $\theta = 60^\circ$

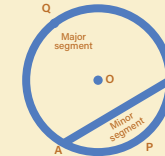
We know that

$$\begin{aligned} \text{Area of sector of circle} &= \frac{\theta}{360^\circ} \times \pi r^2 \\ &= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2 \\ &= \frac{132}{7} = 18.8\text{cm}^2 \end{aligned}$$



Segment

-The area made by an arc and a chord is called the segment of the circle.



Major segment

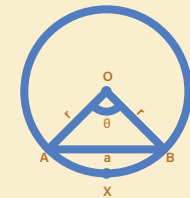
-The other part of the circle except for the area of the minor segment is called a Major segment.

Area of Major segment = Area of circle - Area of minor segment.

Note: Area of circle = Area of minor segment + Area of major segment.

Minor segment

-The area made by chord AB and arc A x B is the minor segment.



Area of Minor segment = Area of minor sector -

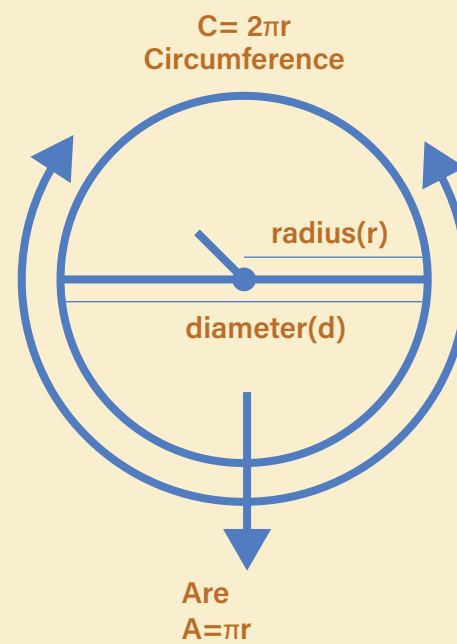
$$\text{Area of triangle ABO} = \frac{\theta}{360^\circ} \times \pi r^2 - \frac{r^2}{2} \sin \theta$$

PERIMETER AND AREA OF CIRCLE

-Area of circle = πr^2 .

-Circumference of circle: The perimeter of a circle is the distance covered by going around its boundary once.

Perimeter of circle = $2\pi r$



where, $\pi = \frac{22}{7}$ or 3.14 and r is the radius of the circle.

SURFACE AREAS AND VOLUMES

CONVERSION OF UNITS

UNIT OF LENGTH

- 1cm = 10mm
- 1dm = 10cm = 100mm
- 1m = 10dm = 100cm = 1,000mm
- 1dam = 10m = 1,000cm
- 1hm = 10dam = 100m
- 1km = 1,000m = 100dam = 10hm

UNIT OF AREA

- 1cm² = 10mm x 10mm = 100mm²
- 1dm² = 10cm x 10cm = 100cm²
- 1m² = 100cm x 100cm = 10,000cm²
- 1dam² = 10m x 10m = 100m²
- 1hm² = 100m x 100m = 10,000m²
- 1km² = 10hm x 10hm = 100hm²

UNIT OF VOLUME

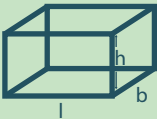



- 1cm³ = 1,000mm³; 1dm³ = 1,000cm³
- 1m³ = 1,000dm³ = 1,000litres
- 1litre = 1dm³ = 1,000cm³;
1kl = 1,000l = 1m³
- 1ml = 1cm³

SURFACE AREA AND VOLUME OF COMBINATION OF SOLIDS

-To find the volume of combination of two or more basic solids like a cone, a cylinder, a sphere, a hemisphere, a cube or a cuboid etc., we add their volumes.

-To find the surface area of combination of two or more basic solids, it is not necessary to add their surface areas directly.

SOLIDS

S.No	Name and Figure	Lateral/Curved Surface Area	Total Surface Area	Volume
1.	Cuboid 	$2h (L+B)$	$2(lb+bh+hl)$	lhb
2.	Cube 	$4l^2$	$6l^2$	l^3
3.	Right Circular Cylinder 	$2\pi rh$	$2\pi r(h+r)$	πr^2h
4.	Right Circular Cone 	πrl	$\pi r(l+r)$	$\frac{1}{3} \pi r^2h$
5.	Sphere 	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
6.	Hemisphere 	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
7.	Hemispherical Shell 	External= $2\pi R^2$ Internal= $2\pi r^2$	$3\pi R^2+\pi r^2$ $=\pi(3R^2+r^2)$	$\frac{2}{3}\pi(R^3-r^3)$

STATISTICS

Mean

-It is defined as the sum of the values of all the observation divided by the total number of observations. The mean of x of the data is given by:

$$\bar{x} = \frac{\sum i^n f_i x_i}{\sum i^n f_i} = \frac{\sum f_i x_i}{\sum f_i}$$

where, f_i is the frequency of i^{th} observation x , and i varies from 1 to n .

Class mark=

$$= \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

MEAN OF GROUPED DATA

MODE OF GROUPED DATA

Mode

$$\text{Mode} = l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$$

where, l = lower limit of the modal class
 h = size of the class intervals'

f_1 frequency of the modal class

f_0 =frequency of the class preceeding the modal class

f_2 = frequency of the class succeeding the modal class.

Eg:

The following table shows the ages of the patient admitted in a hospital during a year.

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of patient	6	11	21	23	14	5

Find the mode of the data given above.

Age	Number of patient
5-15	6
15-25	11
25-35	21 f_0
35-45	23 f_1
45-55	14 f_2
55-65	5

$$\text{Mode} = l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$$

Modal class= Interval with highest frequency
 = 35-45

where, l = lower limit of the modal class = 35

h = size of class intervals = 15-5= 10

f_1 = frequency of the modal class = 23

f_0 =frequency of the class preceeding the modal class = 21

f_2 = frequency of the class succeeding the modal class = 14

$$\text{Mode} = 35 + \frac{(23 - 21)}{2(23) - 21 - 14} \times 10 = 36.8$$

EMPIRICAL FORMULA

-The three measures of central tendency are Mean, Median and Mode which are related as $3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$

MEDIAN OF GROUPED DATA

-For ungrouped data, we first arrange the data values of the observation in ascending order.
 If n is odd, the median is the $(n+1/2)^{\text{th}}$ observation.

If n is even the median will be average of the $n/2^{\text{th}}$ and the $(n/2 + 1)^{\text{th}}$ observation.

-Median Class: Class with cumulative frequency just greater than half of sum of frequencies.

- For grouped data, we use the following formula for calculating the median.

$$\text{Median} = l + \frac{(n/2 - cf)}{f} \times h$$

where, l = lower limit of median class

n = number of observations

cf = cumulative frequency of class preceeding the median class

f = frequency of median class

h = class size (assuming class size to be equal

Modal Class

-The class with the maximum frequency.

Method of Finding Mean

Direct Method

Eg.

A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of Plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of Houses	1	2	1	5	6	2	3

Using direct Method

Number of Plants	Number of Houses (f_i)		$f_i x_i$
0-2	1	$0+2/2=1$	$1 \times 1=1$
2-4	2	$2+4/2=3$	$2 \times 3=6$
4-6	1	$4+6/2=5$	$1 \times 5=5$
6-8	5	$6+8/2=7$	$5 \times 7=35$
8-10	6	$8+10/2=9$	$6 \times 9=54$
10-12	2	$10+12/2=11$	$2 \times 11=22$
12-14	3	$12+14/2=13$	$3 \times 13=39$
	$\Sigma f_i = 20$		$\Sigma f_i x_i = 162$

Mean = $\Sigma f_i x_i / \Sigma f_i = 162/20 = 8.1$ plants.

Assumed Mean Method

Eg.

The following table gives the information about the marks obtained by 110 students in an examinations.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	1	2	1	5	6

Find the mean marks of the student using assumed mean method.

Class	Frequency	Class Marks (x_i)	$d_i = x_i - a$	$f_i d_i$
0-10	12	5	$5-25 = -20$	-240
10-20	28	15	$15-25 = -10$	-280
20-30	32	25-o	$25-25 = 0$	0
30-40	25	35	$35-25 = 10$	250
40-50	13	45	$45-25 = 20$	260
Total	$\Sigma f_i = 110$			$\Sigma f_i d_i = -10$

Assumed mean = $a = 25$

Mean = $x = a + \Sigma f_i d_i / \Sigma f_i$

$$= 25 + (-10/110)$$

$$= 24.9$$

hence, the mean marks of the students are 24.9.